A STUDY OF A FLAPPER-NOZZLE AMPLIFIER

by

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NOMENCLATURE

A	cross-sectional area
h	enthalpy
k	ratio of specific heats
ρ	density
P	pressure
p	small increment of pressure
T	temperature, $^\circ$ R.
V	velocity
θ	time
v	volume
X	gap distance
x	small increment of X
Y	out-put displacement
y	small increment of out-put displacement
X(s)	Laplace transform of x
Y(s)	Laplace transform of y

I. INTRODUCTION

A control system usually includes power-amplifying equipment because the power required to vary the controlled quantity is large compared to the power available in the reference input. In both pneumatic and hydraulic control systems, the flapper valve (or flapper amplifier) is a most common and basic device for high gain and power-amplifying.

Generally speaking, pneumatic systems have the following advantages over hydraulic systems (6)*:

1. Availability of working medium. In a pneumatic system, air can be vented to the atmosphere and is readily available at most places, therefore no return line is necessary. In hydraulic systems, a return line must be used to provide venting.

2. Reliability. Pneumatic systems are clean, inexpensive and relatively trouble free. Hydraulic systems are dirty because of its working medium, dirt being a source of trouble. Therefore, an important advantage of pneumatic systems is that they are highly reliable.

3. In most hydraulic systems, petroleum-base fluids are preferred for their anti-corrosive and lubricating qualities. These fluid are flammable and create a fire hazard. While in pneumatic systems, there is no danger of fire hazard.

Now, consider the conventional flapper valve shown in Fig. 1. When the flapper is closed so that there is no flow, the controlled pressure rises to the stagnation pressure of the supply air. As the flapper opens, the controlled pressure will be reduced and will approach the ambient pressure. The pressure in the bellows is the quantity to be controlled and the gap-distance

* Numbers in parentheses refer to the items of references.

is the controlling variable. It is easily seen that as the gap-distance decreases, the flow rate also decreases. If the supply pressure, P_s, is constant, then, the flow rate is low if P_c is high and vice versa. Since the flow rate is controlled by gap-distance X, the load pressure P_c is also controlled by X. Fig. 2 (I) shows a typical relation between P_c and X for the conventional valve shown in Fig. I. As X increases, P_c approaches P_a, and as X approaches zero, P_c goes to the stagnation pressure of supply air.

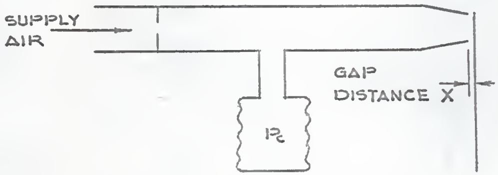


Fig. I

There is a point of inflection (the point at which $\frac{dP}{dX}$ does not increase nor decrease). Near this point, the curve can be adequately approximated by

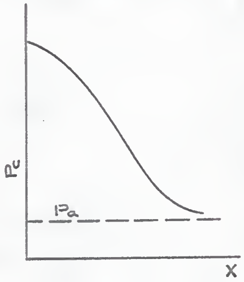


Fig. 2

a straight line. For example, part ab in Fig. 2 is nearly a straight line and hence pressure and gap-distance are linearly related along ab.

As shown in Fig. 2, P_c is always above P_a, the ambient pressure, therefore the bellows must be spring-loaded, allowing operation about some positive pressure point such as point b. Notice that a spring is needed simply because the pressure is always positive. If an operating point of zero gauge pressure can be found ($=P_a$), then there is no need for the spring. When the pressure is positive, it expands the bellows; when the pressure is negative, the atmospheric pressure will return the bellows to its original position. In order to accomplish this, it is necessary to produce a partial vacuum which varies with the flow rate (hence controlled by Gap-Distance).

In this report, an effort was made to investigate the response of a "Venturi Tube" fluid amplifier which reduces the controlled pressure below the ambient pressure under some conditions using supply pressures smaller than normal. Venturi Tube details will be discussed later.

II. STEADY STATE OPERATION OF VENTRUI TUBE AMPLIFIER

A. Introduction

It is a well known fact that when a flow passes through a region of decreasing cross-sectional area, the velocity increases and the pressure decreases. Consider the passage shown in Fig. 3. For purposes of illustration, assume incompressible flow, i.e., $\rho_1 = \rho_2$. From the continuity

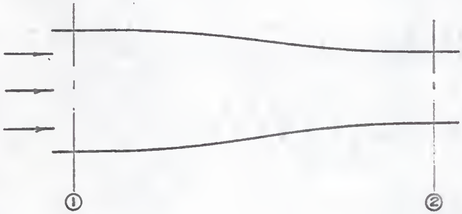


Fig. 3

equation, the following relation is obtained,

$$\rho_1(A_1V_1) = \rho_2(A_2V_2) \qquad \text{or} \qquad A_1V_1 = A_2V_2$$

Since $A_1 > A_2$, $\therefore V_1 < V_2$. Next, consider the energy equation for incompressible, steady and frictionless flow (Bernoulli equation),

$$P_1/\rho_1 g + V_1^2/2g = P_2/\rho_2 g + V_2^2/2g \qquad \text{where } \rho_1 = \rho_2$$

It is easily seen as V_2 increases, P_2 has to decrease in order to keep the sum constant. If P_1 is a little higher than the atmospheric pressure, and

$V_2 \gg V_1$, then P_2 could be lower than atmospheric pressure.

In applying the Venturi Tube principle explained above, the new amplifier has the arrangement shown in Fig. 4.

B. Steady State Analysis

In analyzing the flapper amplifier shown in Fig. 4, a few restrictions and assumptions were made:

1. The flow is assumed to be isentropic and one-dimensional.
2. The perfect gas laws are obeyed. (i.e., $P = \rho RT$ and C_p = constant)
3. The effect of viscosity is negligible.

In addition, for the flapper valve shown in Fig. 4, the exit circumference is considered equivalent to the exit plane of a convergent nozzle, as shown in Fig. 5.

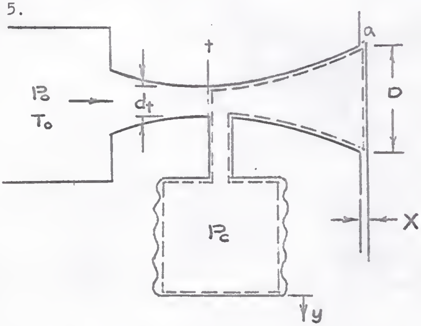


Fig. 4

Referring to Fig. 4, the pressure at section a is assumed to be constant and equal to or greater than the ambient pressure for a fixed stagnation pressure. In other words, P_a is independent of gap-distance X. Inasmuch as an isentropic process is considered, ρ_a and V_a, the density and veloc-

city, respectively, of air flow at section a are also constant for constant stagnation pressure and temperature. It is also reasonable to assume $X \ll D$ for all values of X. Let us define:

P_o = stagnation pressure of supply air

T_o = stagnation temperature of supply air

P_c = pressure to be controlled

( )* = signifies critical state*

X* = value of X corresponding to critical state at section t, as shown in Fig. 4.

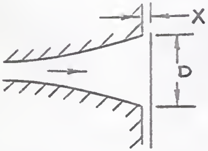
The continuity equation gives a relation between section t and a for steady flow:

$$\rho_t(\pi/4)\, d_t^2 V_t = \rho_a(\pi D X) V_a \quad \text{or}$$

$$X = \frac{\rho_t V_t d_t^2}{4 \rho_a V_a D} \qquad (1)$$

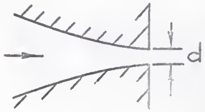
Under critical condition, eq. (1) becomes

$$X^* = \frac{\rho_t^* V_t^* d_t^2}{4 \rho_a V_a D} \qquad (1a)$$



AREA OF CIRCUMFERENCE

= πDX



EQUIVALENT CONVERGENT

NOZZLE, WHERE $\frac{\pi}{4} d^2 = \pi DX$

Fig. 5

*The critical state is the state at which the flowing velocity equals the sonic speed at that state.

If P_o and T_o are fixed, then, $*$ and V_t^* can be calculated. In other words, when supply air has constant state, X^* will depend on d_t and D only. The gain of an amplifier is defined as $\Delta P/\Delta X$. It is seen that making d_t small and D large gives us a smaller value of X^*, hence a greater gain can be attained.

By writing the energy equation, the following relation between sections t and a is obtained:

$$h_t + V_t^2/2gJ \quad = \quad h_a + V_a^2/2gJ \tag{2}$$

Notice $h_t = C_p T_p$, $h_a = C_p T_a$ and $T_t/T_a = (P_t/P_a)^{\frac{k-1}{k}} = (P_t/P_a)^{0.2857}$

Solve for V_t^2/V_a^2 from eq. (2):

$$V_t^2/V_a^2 = 1 - \frac{2gJC_pT_a}{V_a^2}\left[(P_t/P_a)^{0.2857} - 1\right] \tag{3}$$

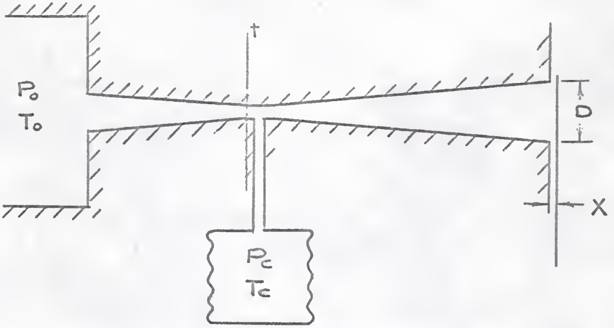
or

$$V_t^2/V_a^2 = 1 - 12025\ (T_a/V_a^2)\left[(P_t/P_a)^{0.2857} - 1\right] \tag{3a}$$

Combining equations (1) and (3a) and keeping $(\rho_t/\rho_a) = (P_t/P_a)^{1/k}$ in mind, the following relation is obtained:

$$X = (P_t/P_a)^{1/k}(d_t^2/4D)\left\{1 - 12025(T_a/V_a^2)\left[(P_t/P_a)^{0.2857} - 1\right]\right\}^{\frac{1}{2}} \tag{4}$$

Eq. (4) shows X as a function of (P_t/P_a) or (P_c/P_t). Fig. 6 shows a numerical example. It is seen that there is no inflection point as there is in Fig. 2. But, as X goes closer to X^*, the gain approaches infinity. Fig. 7 shows the same relation in dimensionless form for different values of P_a/P_o.



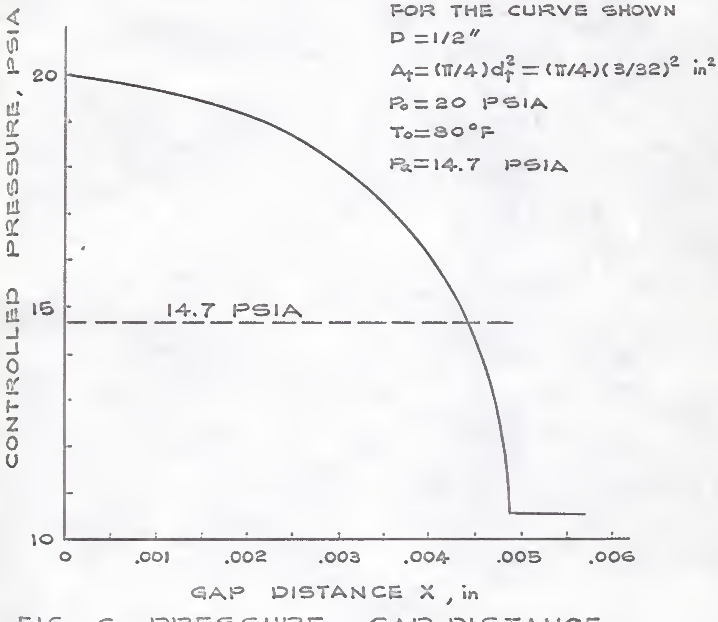


FIG. 6 PRESSURE – GAP DISTANCE RELATION

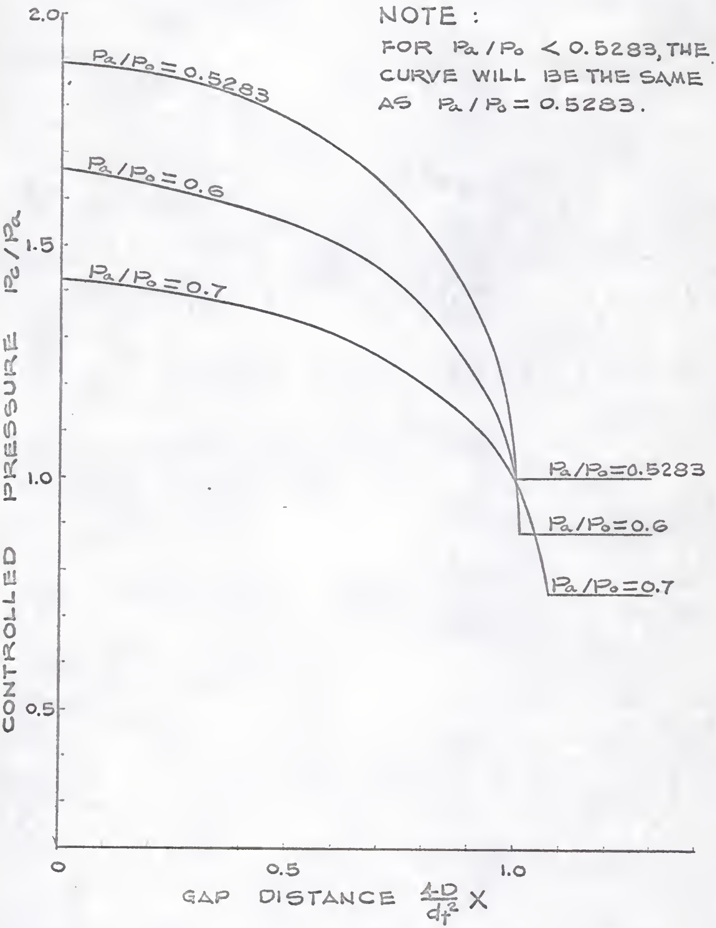


FIG. 7 NORMALIZED PRESSURE - GAP DISTANCE RELATION

Fig. 7 tells us that on the range $\frac{4D}{d_t} X < 0.5$ (when P_c is high) P_c and X have a better linear relation, but the gain is low.

As mentioned previously, when the flapper amplifier is operated around the point $P_c = P_a$, the bellows does not have to be spring loaded. This is the most important advantage by using a convergent-divergent restriction.

C. Experimental Data

A laboratory set-up of a two-dimensional flapper amplifier with 1/64" depth is shown in Fig. 8.

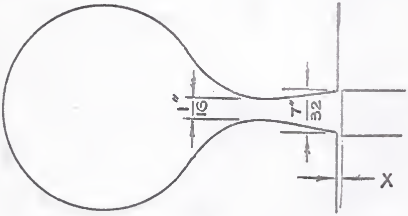


Fig. 8

The data obtained for P_a/P_o = 0.6 and 0.7, respectively, are tabulated below and plotted as shown in Fig. 9.

$P_a/P_o = 0.6$															
1000X in	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0
P_c in Hg	19.10	18.85	18.35	16.87	14.30	9.55	7.30	3.45	1.50	-1.05	-2.10	-2.85	-3.15	-3.45	-3.45

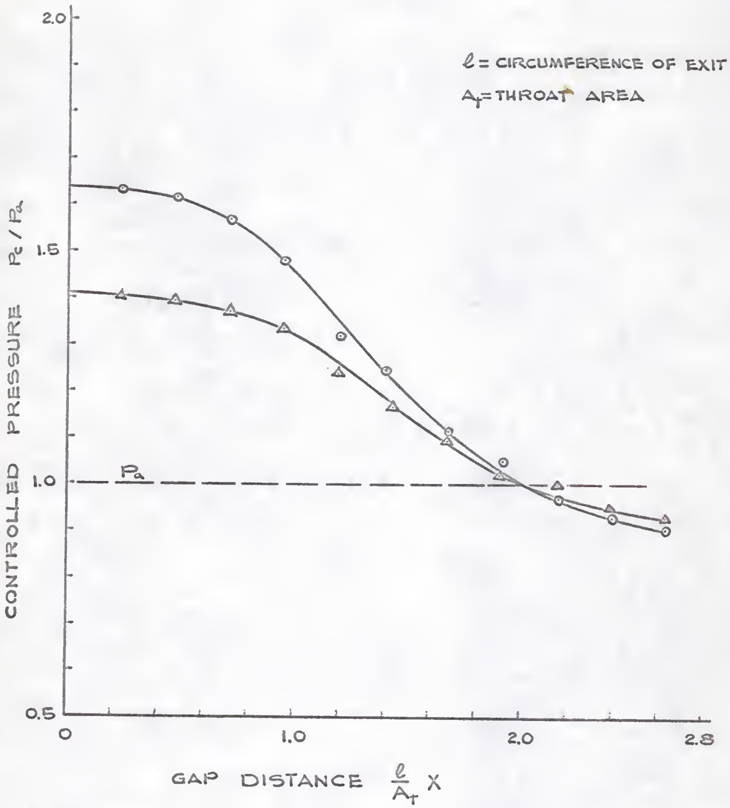


FIG. 9 PRESSURE—GAP DISTANCE RELATION —EXPERIMENTAL DATA

$P_a / P_o = 0.7$															
1000 X in	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0
P_c in Hg	12.25	12.15	11.85	11.20	9.90	7.35	5.10	2.85	1.15	0	-1.50	-1.85	-2.15	-2.35	-2.40

A comparison of mathematical analysis and data obtained experimentally shows that the latter deviates from the former at lower pressures and the values of X* are greater than those calculated from the mathematical formula. The reason might be the effects of viscosity and high degree of turbulence.

The effect of viscosity reduces effective area so that πDX can not represent the real cross-sectional area. In order to make the flow rate larger so that the properties at the throat becomes critical, the cross-sectional area πDX^* has to be larger. This might be the reason why X* from experimental data is greater.

On the other hand, when the flow rate increases, the friction or turbulence also increases. This makes the assumption of isentropic flow less accurate.

Fig. 9 shows that there is an inflection point near $P_c = P_a$. This means a very good linearrelation between P_c and X can be obtained near $P_c = P_a$.

III. THE TRANSFER FUNCTION OF VENTURI-TUBE AMPLIFIER

A. Linearized Operation

Assume V is a nonlinear function of U and let the nonlinear relation between the two variables U and V be shown in Fig. 10.

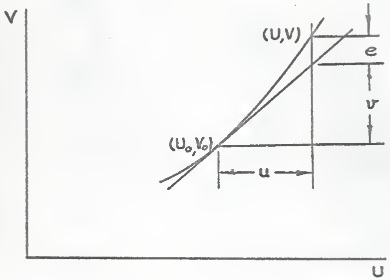


Fig. 10

If the initial point is (U_o, V_o) and the abcissa of the final point is U, then, the abscissa intersects the nonlinear function at V, and it intersects the tangent at $(V_o + v)$. From Fig. 10, the equation for V is as the following:

$$V = V_1 + v + e = V_1 + v$$

Now, the slope of the tangent at (U_o, V_o) is

$$\left(\frac{dV}{dU}\right)_o = \frac{v}{u}, \quad \text{or} \quad v = u \left(\frac{dV}{dU}\right)_o$$

Therefore, the new ordinate can be approximated, within a small range of U, by

$$V = V_1 + \left(\frac{dV}{dU}\right)_o u$$

The last equation explains the theory of small perturbation, and the procedure is known as linearization.

B. The Transfer Function

In considering the dynamic operation of the flapper amlifier, it is helpful to use the continuity equation for unsteady flow:

Mass Flow Rate in = Mass Flow Rate out + Rate of Mass Stored

In applying the above equation, it is convenient to choose a control volume. The boundary of the chosen control volume is shown by the dotted line in Fig. 4.

Mass Flow Rate in = $\rho_t(\pi/4)d_t^2V_t$

Mass Flow Rate out = $\rho_a(\pi DX)V_a$

Rate of Mass Stored = $\frac{dM}{d\theta} = \frac{dM}{dP_c}\frac{dP_c}{d\theta} = Q.\,\dot{P}_c$

where

M = mass in the control volume

$Q = \frac{dM}{d\theta}$ = capacitance of the control volume (5)

For convenience, let us consider a unit capacitance. Then, the continuity equation becomes:

$$\rho_t(\pi/4)d_t^2V_t = \rho_a(\pi DX)V_a + \dot{P}_c \tag{5}$$

Now, refer to Fig. 4 again. Let A_c be the cross-sectional area of the bellows and y be a small increase of the length of the bellows, then

$\Delta P_c A_c = Ky$

or

$$\frac{d(\Delta P_c)}{d\theta} = \frac{dP_c}{d\theta} = \frac{K}{A_c}\frac{dy}{d\theta} = \frac{K}{A_c}\dot{y}$$

Thus, eq. (5) becomes

$$\rho_t(\pi/4)d_t^2V_t = \rho_a V_a(\pi D)X + \frac{K}{A_c}\dot{y} \qquad (7)$$

The energy equation $C_pT_o = C_pT_t + V_t^2/2gJ$ gives

$$V_t = (2gJC_pT_o)^{\frac{1}{2}}\left[1-(P_t/P_o)^{0.2857}\right]^{\frac{1}{2}} \qquad (8)$$

From the adiabatic relation $P_o/\rho_o^k = P_t/\rho_t^k = C$ (=constant), the expression for ρ_t is obtained:

$$\rho_t = (P_t/C)^{1/k} \qquad (9)$$

The left-hand side of eq. (7) thus has the following expression:

$$\rho_t(\pi/4)d_t^2V_t = (P_c/C)^{1/k}(\pi/4)d_t^2\,(2gJC_pT_o)^{\frac{1}{2}}\left[1-(P_c/P_o)^{0.2857}\right]^{\frac{1}{2}} = Z$$

where Z is a function of P_c only. Notice, $P_t = P_c$ has been assumed.*

Linearize equation (7), we get

$$-Np_c = (\rho_a V_a \pi D)x + \dot{p}_c \qquad (10)$$

or

$$-N\frac{K}{A_c}y = (\rho_a V_a \pi D)x + \frac{K}{A_c}\dot{y} \qquad (10a)$$

where $-N = \frac{dZ}{dP_c}$

$$= (\pi/4)d_t^2(2gJC_pT_o)^{\frac{1}{2}}(\rho_o/P_o)\left[\frac{0.143}{1-(P_c/P_o)^{0.2857}} - \frac{1}{k}(P_c/P_o)^{-0.2857}\right]$$

$$\times\left[1-(P_c/P_t)^{0.2857}\right]^{\frac{1}{2}}$$

Transforming eq.(10a) into the Laplace domain:

* For more details about this assumption, see "DISCUSSION AND SUMMARY".

$$\frac{Y(s)}{X(s)} = - \frac{A_c\,(V_a D)}{KN} \quad \frac{1}{\frac{1}{N}s + 1} \tag{11}$$

Eq. (11) is the transfer function of the Venturi-Tube flapper amplifier and has the same form as that of the conventional flapper amplifier (1).

IV. DISCUSSION AND SUMMARY

1. From the normalized pressure gap-distance relation shown in Fig. 7 it is seen that as $X > X^*$, the gap-distance has no effect on the controlled pressure, and the gain becomes zero. P_a/P_o will never be smaller than 0.5283. Hence the term "ambient pressure" means atmospheric pressure when $P_o \leq 27.85$ psia (=14.7/0.5285), and it will be higher than atmospheric pressure if $P_0 > 27.85$ psia.

2. If is the time constant of the amplifier, eq. (11) gives

$$\tau = 1/N \tag{12}$$

Eq. (12) shows that the time constant per unit capacitance varies inversely with N. The curve showing normalized 1/N versus P_c was plotted in Fig. 11. The time constant approaches zero as P_c goes to P_o and it becomes infinity when P_c/P_o approaches 0.5283. Notice that

$$\frac{1}{N} = \frac{\text{the value read from Fig. 11}}{\dfrac{d_t^2 \, (2gJC_pT_o)^{\frac{1}{2}}\pi}{4P_o}}$$

A smaller value of 1/N can be attained by making d_t large. (hence the flow rate becomes high)

3. The gain constant keeps changing as the operating point moves since this is a nonlinear device. Let G denote the gain constant, defined as:

$$G = \frac{\pi V_a A_c P_a}{K} \frac{D}{N} \tag{13}$$

Therefore, G is proportional to D and 1/N. Since 1/N is proportional to $1/d_t^2$ (see the middle of this page), a high gain constant is obtained when D is large and d_t, small. If a high gain constant and a small time constant

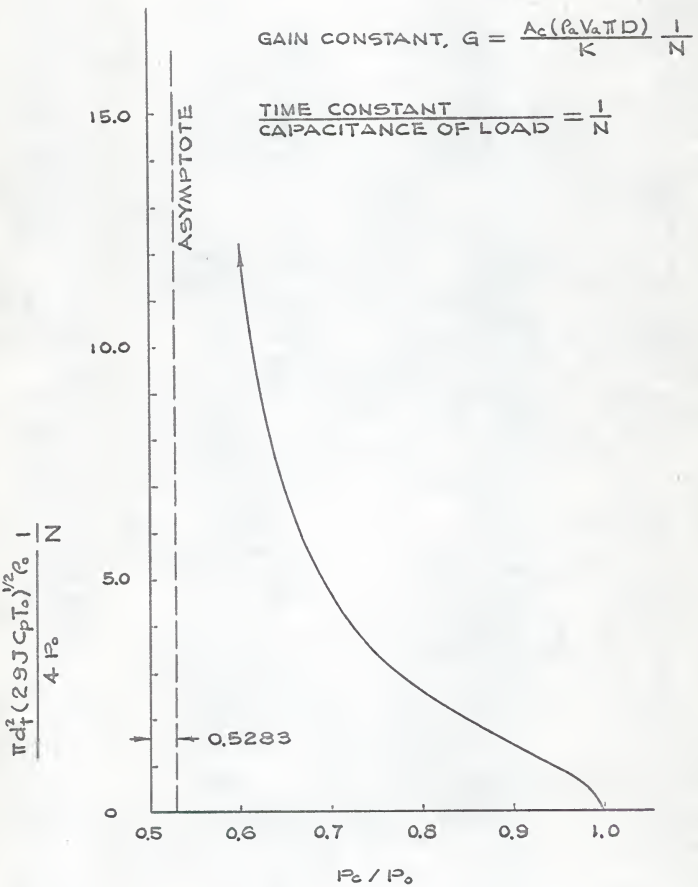


FIG. 11 GAIN CONSTANT & TIME CONSTANT V.S. P_c/P_o

are to be attained simultaneously, D/d_t and d_t must be large which requires a high air flow rate. These two requirements must be compromised to meet the specifications of a particular case.

4. Refer to Fig. 12. If a contoured plug is used instead of a simple flat flapper, the pressure gap-distance relation can be improved. For example, by contouring the plug such that the relation between X and the "effective" exit area is similar to that shown in Fig. 13, the curves in Fig. 7 theoretically can be made linear. The contour required is dependent on the supply air properties and the ambient pressure.

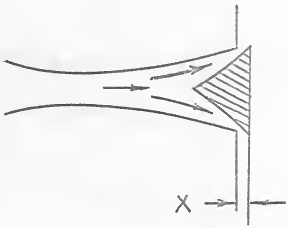


Fig. 12

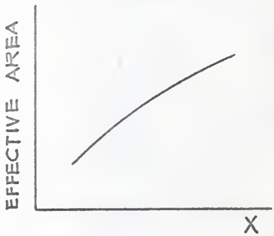


Fig. 13

5. In analyzing the flapper valve-volume (or load) combination, three different cases can be assumed, according to reference 4.

(a) The Isentropic Model

i. The volume of load is separated from the valve by an impermeable membrane. The membrane will not support a pressure difference and will not allow any heat transfer between the volume and the valve.

ii. The gas inside the volume behaves isentropically.

(b) The Perfect Mixing Model

By the statement of perfect mixing, the author of reference 4 means that the stagnation enthalpy of the fluid leaving the exit of the flapper amplifier is the same as the stagnation enthalpy of the fluid in the load (or the volume).

(c) The Imperfect Mixing Model

In this case, charging and discharging the volume are distinguished.

i. In charging the volume, it is assumed that the stagnation enthalpy of the fluid leaving the exit of the flapper amplifier is the same as that of the fluid flowing into the flapper amplifier.

ii. In discharging the volume, two assumptions are made:

(A) The stagnation enthalpy of the fluid leaving the flapper amplifier is a weighted average of the stagnation enthalpies of the flow into the amplifier and the flow leaving the volume.

(B) The ratio of the stagnation temperature of the fluid leaving the amplifier to the supply stagnation temperature is close to unity. (0.9 to 1.1)

Numerically, there is really very little difference between the three models. The reason is that the temperature ratio (the temperature in the volume/supply temperature) ranges only between 0.9 and 1.1, and the variation in temperature ratio has a small effect on the pressure. For an calculations, then, one might as well use the isentropic model, since it is the simplest to work with.

In this report, the isentropic model is adopted.

- end -

ACKNOWLEDGEMENT

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AN ABSTRACT OF A MASTER'S REPORT

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ABSTRACT

In this report, a proposed method to eliminate the return spring of a conventional flapper amplifier was studied. A Venturi Tube was employed to produce a partial vacuum so that an operating point of zero gauge pressure could be attained. Thus, the spring force is replaced by the atmospheric pressure.

The steady state operation of the new flapper amplifier was investigated and the transfer function was derived. The relation between the controlled pressure and gap distance was determined; equations for the controlled pressure and time constant (and gain constant) relations were derived also. The results show that a high gain constant and small time constant can be attained simultaneously, but the flow rate must be high. Some experimental data were obtained.